Quantifying Deformations in 2D Flowing Foams

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In a continuum description of materials, the stress tensor field, $\overline{\overline{\sigma}}$, quantifies the internal forces the neighboring regions exert on a region of the material. The classical theory of elastic solids assumes that stress determines the strain, while hydrodynamics assumes that stress determines the strain rate. To extend both successful theories to more general materials, which display both elastic and fluid properties, we introduce a descriptor generalizing the classical strain to include plastic deformations: the "statistical strain", based on averages on microscopic details. We apply such a statistical analysis to a two-dimensional foam steadily flowing through a constriction, a problem beyond reach of both theories, and prove that the foam has the elastic properties of a *linear and isotropic* continuous medium.

A "plastic" deformation means that microscopic rearrangements take place in the material, so that the microscopic pattern does not return to its initial condition even after the applied force has ended. An example is a two-dimensional foam steadily flowing through a constriction (Fig. 1). This apparently simple example is utterly intractable from the perspective of both elasticity theory and Navier-Stokes treatments. Here we present a new approach to analyze complex flows of disordered materials.

The foam is prepared by blowing air into the bottom of a column of soap solution. Bubbles float to the top of the solution and enter a horizontal channel. The channel is made of two parallel Plexiglas plates 0.5 mm apart. A 5-mm wide constriction near the end of the channel disrupts the otherwise homogeneous flow (Fig. 1).

To measure the Eulerian velocity field, we track the center of mass of each bubble between two successive images. The velocity field appears smooth and regular, qualitatively indicating that the foam behaves as a continuous medium. Stress in the foam has dissipative and elastic components; the pressure inside the bubbles and the network of bubble edges contribute to the latter. Since the pressure stress is isotropic, it does not contribute to the elastic normal stress difference $\overline{\overline{\sigma}}_{xx} - \overline{\overline{\sigma}}_{yy}$ or shear stress $\overline{\overline{\sigma}}_{xy}$, which is thus entirely due to the network structure. We measure locally the network stress in each mesoscopic volume $\mathcal{V}$ centered around the point of measurement, of a size that is larger than a bubble but much smaller than the channel width. We proceed as follows: 1. identify the bubble edges which cross the boundaries of the volume, 2. determine the tension of each edge, 3. determine the average force $\vec{f}$ on a boundary element $d\vec{S}$ by vectorially adding these tensions and obtain $\overline{\overline{\sigma}}$. Fig. 2 clearly shows that the stress field is heterogeneous, where the upstream influence of the constriction becomes visible as $\overline{\overline{\sigma}}_{xx} - \overline{\overline{\sigma}}_{yy}$ changes sign.

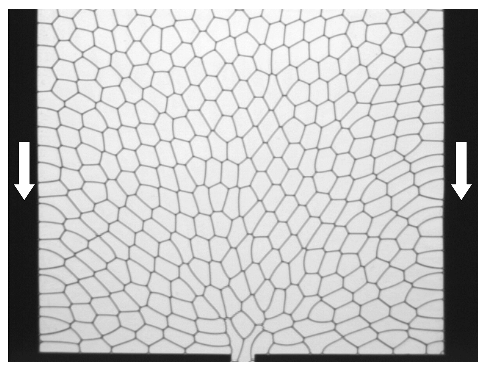

Fig 1. Two-dimensional foam flowing through a constriction. The 10 cm wide field of view shows only the end of the 1 m long horizontal channel.

We list all vectors $\vec{\ell}$ that link the two vertices connected by one bubble edge, from which we construct a tensor $\vec{\ell} \otimes \vec{\ell} = (\ell_i \ell_j)$. This tensor averaged over volume $\mathcal{V}$ defines the local *t*exture tensor $\overline{\overline{M}}$: $M_{ij} = \langle \ell_i \ell_j \rangle_{\mathcal{V}},$. This symmetric tensor has two positive eigenvalues, the larger lies in the direction of bubble elongation. It reflects at large scales the relevant features of the actual microstructure of the material. For instance, Fig. 3 shows an example of $\overline{\overline{M}}$ measured in the flowing foam experiment, which quantifies the qualitative impression of compression or elongation we ob-

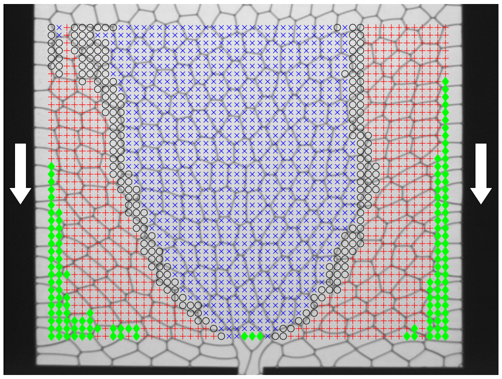

Fig 2. Snapshot of the foam with regions colored according to the sign of the experimental value of the normal elastic stress difference $\overline{\overline{\sigma}}_{xx} - \overline{\overline{\sigma}}_{yy}$: blue $\times$, negative; black $\circ$, zero within error; red $+$, positive; green $\diamond$, values we omit for analysis.

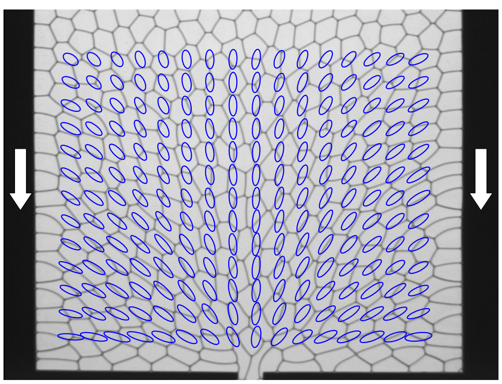

Fig 3. Experimental measurement of $\overline{\overline{M}}$, superimposed on a snapshot of the foam (Fig. 1). We display $\overline{\overline{M}}$ as an ellipse, the length of each axis (in arbitrary units, the same scale for each ellipse) being proportional to an eigenvalue. We omit data in boxes that touch the channel wall.

tain by looking at Fig. 1.

We define the "statistical strain tensor":

$$\overline{\overline{U}} = \frac{1}{2}\left(\log \overline{\overline{M}} - \log \overline{\overline{M}}_0\right).$$

Here the reference value $\overline{\overline{M}}_0$ is chosen in the undeformed, isotropic foam far upstream of the constriction (a choice that plays no role in the elastic properties). The "statistical strain" tensor reduces to the usual definition of strain in the validity limits of classical elasticity. This tensor is purely geometric, and does not explicitly depend on stresses and forces. It applies to a whole general class of materials with both elasticity and plastic rearrangements, whenever we can experimentally measure the relevant information: whether in detail (list of microscopic positions and links) or as mesoscopic averages. It is a state variable, constant in a steady flow but not necessarily homogeneous, allowing a thermodynamic description of non-equilibrium complex fluids.

How do the elastic stress and the statistical strain relate? We plot the normal stress difference $\overline{\overline{\sigma}}_{xx} - \overline{\overline{\sigma}}_{yy}$ versus $\overline{\overline{U}}_{xx} - \overline{\overline{U}}_{yy}$. Each data point is a measurement derived from averages at one position of the foam. All data points fall on a narrow straight line. Since $\overline{\overline{M}}$, and hence $\overline{\overline{U}}$, is completely independent of $\overline{\overline{\sigma}}$, the high correlation between them reflects the physical constitutive relation required to treat the foam as a continuous medium, in which details of the microstructure appear only through mesoscopic averages. Since different applied strains $\overline{\overline{u}}_{appl}$ can correspond to the same $\overline{\overline{\sigma}}$, such relation does not appear in classical $\overline{\overline{\sigma}}$ *vs.* $\overline{\overline{u}}_{appl}$ plots. Moreover, the relation between $\overline{\overline{U}}$ and $\overline{\overline{\sigma}}$ is linear over the whole range covered by our experiment, the slope of which measures the shear modulus of the foam, which is much beyond the validity regime of classical elasticity. The foam is also nearly isotropic, since we find almost the same value for the *xy* component.

In summary, we have measured the stress, the texture tensor and the statistical strain for a 2D flowing foam. We have shown that the 2D foam behaves like a linear and isotropic continuous material.

Acknowledgements

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References

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